Large-scale Parallel Collaborative Filtering for the Netflix Prize

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Abstract. Many recommendation systems suggest items to users by utilizing the techniques of collaborative filtering (CF) based on historical records of items that the users have viewed, purchased, or rated. Two major problems that most CF approaches have to resolve are scalability and sparseness of the user profiles. In this paper, we describe *Alternating-Least-Squares with Weighted-λ-Regularization* (ALS-WR), a parallel algorithm that we designed for the Netflix Prize, a large-scale collaborative filtering challenge. We use parallel Matlab on a Linux cluster as the experimental platform. We show empirically that the performance of ALS-WR monotonically increases with both the number of features and the number of ALS iterations. Our ALS-WR applied to the Netflix dataset with 1000 hidden features obtained a RMSE score of 0.8985, which is one of the best results based on a pure method. Combined with the parallel version of other known methods, we achieved a performance improvement of 5.91% over Netflix's own CineMatch recommendation system. Our method is simple and scales well to very large datasets.

1 Introduction

Recommendation systems try to recommend items (movies, music, webpages, products, etc) to interested potential customers, based on the information available. A successful recommendation system can significantly improve the revenue of e-commerce companies or facilitate the interaction of users in online communities. Among recommendation systems, *content-based* approaches analyze the content (e.g., texts, meta-data, features) of the items to identify related items, while *collaborative filtering* uses the aggregated behavior/taste of a large number of users to suggest relevant items to specific users. Collaborative filtering is popular and widely deployed in Internet companies like Amazon [16], Netflix [2], Google News [7], and others.

The Netflix Prize is a large-scale data mining competition held by Netflix for the best recommendation system algorithm for predicting user ratings on movies, based on a training set of more than 100 million ratings given by over 480,000 users to nearly 18,000 movies. Each training data point consists of a quadruple (user, movie, date, rating) where rating is an integer from 1 to 5. The test dataset consists of 2.8 million data points with the ratings hidden. The goal is to minimize the RMSE (root mean squared error) when predicting the ratings on the test dataset. Netflix's own recommendation system (CineMatch) scores 0.9514 on the test dataset, and the grand challenge is to improve it by 10%.

The Netflix problem presents a number of practical challenges. (Which is perhaps why, as yet, the prize has not been won.) First, the size of the dataset is 100 times larger than previous benchmark datasets, resulting in much longer model training time and much larger system memory requirements. Second, only about 1% of the user-movie matrix has been observed, with the majority of (potential) ratings missing. This is, of course, an essential aspect of collaborative filetering in general. Third, there is noise in both the training and test dataset, due to human behavior – we cannot expect people to to be completely predictable, at least where their feelings about ephemera like movies is concerned. Fourth, the distribution of ratings per user in the training and test datasets are different, as the training dataset spans many years (1995-2005) while the testing dataset was drawn from recent ratings (year 2006). In particular, users with few ratings are more prevalent in the test set. Intuitively, it is hard to predict the ratings of a user who is sparsely represented in the training set.

In this paper, we introduce the problem in detail. Then we describe a parallel algorithm, alternating-least-squares with weighted-λ-regularization. We use parallel Matlab on a Linux cluster as the experimental platform, and our core algorithm is parallelized and optimized to scale up well with large, sparse data. When we apply the proposed method to the Netflix Prize problem, we achieve a performance improvement of 5.91% over Netflix's own CineMatch system.

The rest of the paper is organized as follows: in Section 2 we introduce the problem formulation. In Section 3 we describe our novel parallel Alternative-Least-Squares algorithm. Section 4 describes experiments that show the effectiveness of our approach. Section 5 discusses related work and Section 6 concludes with some future directions.

2 Problem Formulation

Let $R = \{r_{ij}\}_{n_u \times n_m}$ denote the user-movie matrix, where each element r_{ij} represents the rating score of movie j rated by user i with its value either being a real number or missing, n_u designates the number of users, and n_m indicates the number of movies. In many recommendation systems the task is to estimate some of the missing values in R based on the known values.

We start with a low-rank approximation of the user-item matrix R. This approach models both users and movies by giving them coordinates in a low dimensional feature space. Each user and each movie has a feature vector, and each rating (known or unknown) of a movie by a user is modeled as the inner product of the corresponding user and movie feature vectors. More specifically, let $U = [\mathbf{u_i}]$ be the user feature matrix, where $\mathbf{u_i} \subseteq \mathbb{R}^{n_f}$ for all $i = 1 \ldots n_u$, and let $M = [\mathbf{m_j}]$ be the movie feature matrix, where $\mathbf{m_j} \subseteq \mathbb{R}^{n_f}$ for all $j = 1 \ldots n_m$. Here n_f is the dimension of the feature space, that is, the number of hidden variables in the model. It is a system parameter that can be determined by a hold-out dataset or cross-validation. If user ratings were fully predictable and

n_f sufficiently large, we could expect that $r_{ij} = < \mathbf{u_i}, \mathbf{m_j} >, \forall\, i, j$. In practice, however, we minimize a loss function (of U and M) to obtain the matrices U and M. In this paper, we study the mean-square loss function. The loss due to a single rating is defined as the squared error:

$$\mathcal{L}^2(r, \mathbf{u}, \mathbf{m}) = (r - < \mathbf{u}, \mathbf{m} >)^2. \tag{1}$$

Then we can define the empirical, total loss (for a given pair U and M) as the summation of loss on all the known ratings in Eq. (2).

$$\mathcal{L}^{emp}(R, U, M) = \frac{1}{n} \sum_{(i,j)\in I} \mathcal{L}^2(r_{ij}, \mathbf{u_i}, \mathbf{m_j}), \tag{2}$$

where I is the index set of the known ratings and n is the size of I.

We can formulate the low-rank approximation problem as follows.

$$(U, M) = \arg \min_{(U,M)} \mathcal{L}^{emp}(R, U, M). \tag{3}$$

where U and M are real, have n_f coulmns, but are otherwise unconstrained.

In this problem, (Eq. (3)), there are $(n_u + n_m) \times n_f$ free parameters to be determined. On the other hand, the known ratings set I has far fewer than $n_u n_m$ elements, because it is impossible for all but a very few users to view and rate 18,000 movies. [1] Solving the problem Eq. (3) with many parameters (when n_f is relatively large) from a sparse dataset usually overfits the data. To avoid overfitting, a common method appends a Tikhonov regularization [22] term to the empirical risk function (Eq. (4)).

$$\mathcal{L}^{reg}_{\lambda}(R, U, M) = \mathcal{L}^{emp}(R, U, M) + \lambda(\|U\Gamma_U\|^2 + \|M\Gamma_M\|^2), \tag{4}$$

for a certain suitably selected Tikhonov matrices Γ_U and Γ_M. We will discuss the details in the next section.

3 Our Approaches

In this section, we describe an iterative algorithm, Alternative-Least-Squares with Weighted-λ-Regularization (ALS-WR), to solve the low rank approximation problem. Then we develop a parallel implrmrntation of ALS-WR based on a parallel Matlab platform.

3.1 ALS with Weighted-λ-Regularization

As the rating matrix contains both signals and noise, it is important to remove noise and use the recovered signal to predict missing ratings. *Singular Value*

[1] For instance, in the Netflix Prize problem, the number of known ratings is about 1.1% of all possible ratings.

Decomposition (SVD) is a natural approach that approximates the original user-movie rating matrix R by the product of two rank-k matrices $\tilde{R} = U^T \times M$. The solution given by the SVD minimizes the Frobenious norm of $R - \tilde{R}$, which is equivalent to minimizing the RMSE over all elements of R. However, as there are many missing elements in the rating matrix R, standard SVD algorithms cannot find U and M.

In this paper, we use *alternating least squares* (ALS) to solve the low-rank matrix factorization problem as follows:

Step 1 Initialize matrix M by assigning the average rating for that movie as the first row, and small random numbers for the remaining entries.
Step 2 Fix M, Solve U by minimizing the objective function (the sum of squared errors);
Step 3 Fix U, solve M by minimizing the objective function similarly;
Step 4 Repeat Steps 2 and 3 until a stopping criterion is satisfied.

The stopping criterion we use is based on the observed RMSE on the probe dataset. After one round of updating both U and M, if the difference between the observed RMSEs on the probe dataset is less than 1 bps [2], the iteration stops and we use the obtained U, M to make final predictions on the test dataset. The probe dataset is provided by Netflix, and it has the same distribution as the hidden test dataset.

As we mention in Section 2, there are many free parameters. Without regularization, ALS might lead to overfitting. A common fix is to use Tikhonov regularization, which penalizes large parameters. We tried various regularization matrices, and eventually found the following weighted-λ-regularization to work the best, as it never overfits the test data (empirically) when we increase the number of features or number of iterations.

$$f(U, M) = \sum_{(i,j)\in I} (r_{ij} - \mathbf{u}_i^T \mathbf{m}_j)^2 + \lambda \left(\sum_i n_{u_i} \|\mathbf{u}_i\|^2 + \sum_j n_{m_j} \|\mathbf{m}_j\|^2 \right), \quad (5)$$

where n_{u_i} and n_{m_j} denote the number of ratings of user i and movie j respectively. [3] Let I_i denote the set of movies j that user i rated, then n_{u_i} is the cardinality of I_i; similarly I_j denotes the set of users who rated movie j, and n_{m_j} is the cardinality of I_j. This corresponds to Tikhonov regularization where $\Gamma_U = \mathrm{diag}(n_{u_i})$ and $\Gamma_M = \mathrm{diag}(n_{m_j})$.

Now we demonstrate how to solve the matrix U when M is given. A given column of U, say u_i, is determined by solving a regularized linear least squares problem involving the known ratings of user i, and the feature vectors m_j of the

[2] 1 bps equals 0.0001.
[3] The same objective function was used previously by Salakhutdinov et al. [20] and solved using gradient descent. We will discuss more on their approach in Section 5.2.

movies that user i has rated.

$$
\begin{aligned}
&\frac{1}{2}\frac{\partial f}{\partial u_{ki}} = 0, \quad \forall i,k \\
&\Rightarrow \sum_{j\in I_i} (\mathbf{u}_i^T \mathbf{m}_j - r_{ij}) m_{kj} + \lambda n_{u_i} u_{ki} = 0, \quad \forall i,k \\
&\Rightarrow \sum_{j\in I_i} m_{kj} \mathbf{m}_j^T \mathbf{u}_i + \lambda n_{u_i} u_{ki} = \sum_{j\in I_i} m_{kj} r_{ij}, \quad \forall i,k \\
&\Rightarrow \left(M_{I_i} M_{I_i}^T + \lambda n_{u_i} E\right) \mathbf{u}_i = M_{I_i}\ R^T(i, I_i), \quad \forall i \\
&\Rightarrow \mathbf{u}_i = A_i^{-1} V_i, \quad \forall i
\end{aligned}
$$

where $A_i = M_{I_i} M_{I_i}^T + \lambda n_{u_i} E$, $V_i = M_{I_i}\ R^T(i, I_i)$, and E is the $n_f \times n_f$ identity matrix. M_{I_i} denotes the sub-matrix of M where columns $j \in I_i$ are selected, and $R(i, I_i)$ is the row vector where columns $j \in I_i$ of the i-th row of R is taken.

Similarly, when M is updated, we can compute individual m_j's via a regularized linear least squares solution, using the feature vectors of users who rated movie j, and their ratings of it, as follows:

$$
\mathbf{m}_j = A_j^{-1} V_j, \quad \forall j,
$$

where $A_j = U_{I_j} U_{I_j}^T + \lambda n_{m_j} E$ and $V_j = U_{I_j} R(I_j, j)$. U_{I_j} denotes the sub-matrix of U where columns $i \in I_j$ are selected, and $R(I_j, j)$ is the column vector where rows $i \in I_j$ of the j-th column of R is taken.

3.2 Parallel ALS with Weighted-λ-Regularization

We parallelize ALS by parallelizing the updates of U and of M. We are using the latest verstion of Matlab, which allows parallel Matlab computation in which several separate copies of Matlab, each with its own private workspace, and each running on its own hardware platform, collaborate and communicate to solve problems. Each such running copy of Matlab is referred to as a “lab”, with its own identifier (labindex) and with a static variable (numlabs) telling how many labs there are. Matrices can be private (each lab has its own copy, and their values differ), replicated (private, but with the same value on all labs) or distributed (there is one matrix, but with rows, or columns, partitioned among the labs). Distributed matrices are a convenient way to store and use very large datasets, too large to be stored on one processor and its associated local memory. In our case, we use two distributed copies of the ratings matrix R, one distributed by rows (i.e., by users) and the other by columns (i.e., by movies). We will compute distributed, updated matrices U and M. In computing U we will require a replicated version of M, and *vice versa*. Thus, our labs communicate to make replicated versions of U and of M from the distributed versions that are first computed. Matlab's “gather” function performns the inter-lab communication needed for this.

To update M, we require a replicated copy of U, local to each lab. We use the ratings data distributed by columns (movies). The distribution is by blocks of equal numbers of movies. The lab that stores the ratings of movie j will, naturally, be the one that updates the corresponding column of M, which is movie j's feature vector. Each lab computes $\mathbf{m_j}$ for all movies in the corresponding movie group, in parallel. These values are then "gathered" so that every node has all of M, in a replicated array. To update U, similarly all users are partitioned into equal-size user groups and each lab just update user vectors in the corresponding user group, using the ratings data partitioned by rows. The following Matlab snippet implements the procedure of updating M given U:

```
function M = updateM(lAcols, U)
    lamI = lambda * eye(Nf);
    lM = zeros(Nf,Nlm); lM = single(lM);
    for m = 1:Nlm
        users = find(lAcols(:,m));
        Um = U(:, users);
        vector = Um * full(lAcols(users, m));
        matrix = Um * Um' + locWtM(m) * lamI;
        X = matrix \ vector;
        lM(:, m) = X;
    end
    M = gather(darray(lM));
end
```

For the above Matlab code, lAcols is the local copy of R distributed by columns (movies), locWtM is the vector of n_{m_j} for all movies in the partitioned movie group, and Nlm is the number of movies in the movie group. Nf and lambda correspond to n_f and λ, and they are the only tunable parameters of ALS-WR.

The broadcast step is the only communication cost due to using a distributed, as opposed to a shared-memory, algorithm. For our method it takes up less than 5% of the total run time. The algorithm achieves a nearly linear speedup; for $n_f = 100$, it takes 2.5 hours to update M and U once with a single processor, as opposed to 5 minutes with 30 processors). The converged solution (with 30 ALS iterations) can be computed, for 100 hidden factors, in 2.5 hours.

4 Performance for the Netflix Prize Problem

We run our experiment on a 30-processor Linux cluster of HP ProLiant DL380 G4 machines. All processors are Xeon 2.8GHz and every four processors share 6GB of RAM. For each fixed n_f, we run between 10 to 25 rounds of ALS-WF and stop when one round of U, M update improves by less than 1 bps the RMSE score on the probe dataset. The optimal value of λ is determined by trial and error. [4] The test RMSE is obtained by submission to the Netflix prize website[5]. The true values of the test ratings are unknown to us; for model building and parameter tuning, we exclude the probe dataset from the training dataset and use it for testing. The probe dataset is a subset of the training dataset, provided by Netflix, and it consists of 1,408,395 latest ratings in year 2006, while users are sampled uniform random and at most 9 ratings are drawn for each user. The test dataset is hidden by Netflix but the distribution of the test dataset is the same as the distribution of the probe dataset.

4.1 Post-processing

For post-processing of the prediction results, we first apply a global bias correction technique over each prediction solution. Given a prediction P, if the mean of P is not equal to the mean of the test dataset, we can shift all predicted values by a fixed constant $\tau = \text{mean}(\text{test}) - \text{mean}(P)$. The global bias correction technique can be shown to strictly reduce RMSE. Another technique we use is to linearly combine various predictors to obtain a better predictor. For example, given two predictors P_0 and P_1, we can obtain a family of predictors $P_x = (1-x)P_0 + xP_1$, and use linear regression to find x^* minimizing $\text{RMSE}(P_x)$. Therefore we obtain P_{x^*} which is at least as good as P_0 or P_1.

4.2 Experimental Results for ALS

The most important discovery we made is that ALS-WR never overfits the data if we either increase the number of iterations or the number of hidden features. As Figure 1 shows, with fixed n_f and λ, each iteration improves the RMSE score of the probe dataset, and it converges after about 20 rounds. Different λ values give different final score, and we normally need to try a small number of λ values to get a good RMSE score. Figure 2 shows the performance of ALS-WR with fixed λ value and varying number of hidden features (n_f ranges from 2 to 20). For each experiment, ALS-WR interations continue until the RMSE over the probe dataset improves less than 1 bps. From the figure we can tell that the RMSE monotonically decreases with larger n_f, even though the improvement diminishes gradually.

Next we conduct experiments with real submissions using large values of n_f. For ALS with simple λ regularization ($\Gamma_u = \Gamma_m = E$), we obtain a RMSE of 0.9184. For ALS with weighted-λ-regularization, we obtained a RMSE of 0.9114 with $n_f = 50$, 0.9066 with $n_f = 150$. With $n_f = 300$ and global bias correction, we obtain a RMSE of 0.9017; with $n_f = 400$ and global bias correction, a score of 0.9006 was obtained; with $n_f = 500$ and global bias shift, a score of 0.9000

[4] Empirically we found that for fixed n_f, the convergence RMSE score is a convex function of λ, and the optimal value of λ is monotone decreasing with respect to n_f. Based on these observations, we are able to find the best value of λ for each n_f with only 2-3 experiments.

[5] See http://www.netflixprize.com/rules for the detailed rules of the competition.

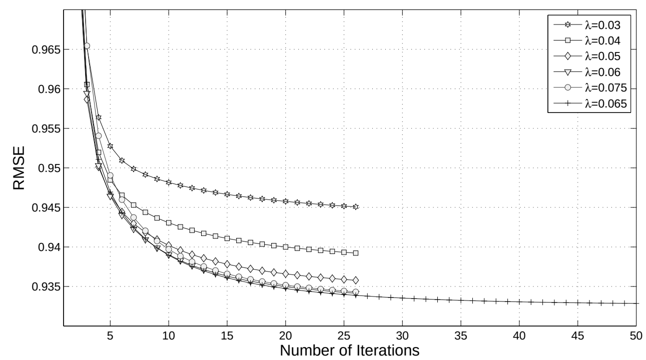

Fig. 1. Comparisons of different λ values for ALS-WR with $n_f = 8$. The best performer with 25 rounds is $\lambda = 0.065$. For this fixed λ, after 50 rounds, the RMSE score still improves but only less than 0.1 bps for each iteration afterwards.

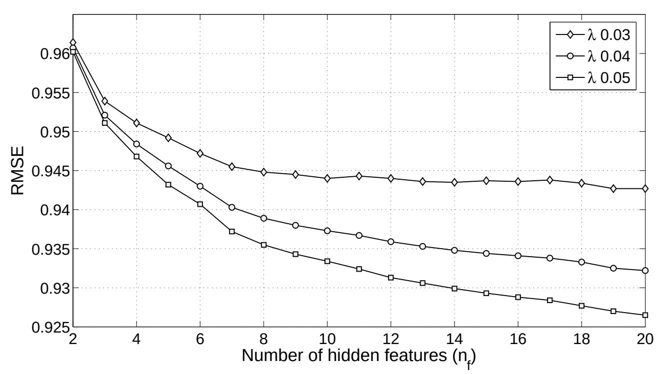

Fig. 2. Performance of ALS-WR with fixed λ and varying n_f.

was obtained. Ultimately, we experimented with $n_f = 1000$ and obtained a RMSE score of 0.8985. [6] Given that 6 bps improvement is obtained from 400 to 500 features, and assuming diminishing (equal-decrease) return with increasing number of features, moving from 500 to 1000 features improves approximately $5 + 4 + 3 + 2 + 1 = 15$ bps. Therefore, 0.8985 is likely the limit we can achieve using ALS with Weighted-λ-Regularization. A RMSE score of 0.8985 translates into a 5.56% improvement over Netflix's CineMatch, and it represents one of the top *single-method* performance according to our knowledge.

4.3 Other Methods and Linear Blending

We also implement parallel versions of two other popular collaborative filtering techniques as described in this section. In each case, the speedup as compared to a single-processor version is roughly a factor of n on a cluster of n processors.

The Restricted Boltzmann Machine (RBM) is a kind of neural network where there are visible states and hidden states, and undirected edges connecting each visible state to each hidden state. There are no connections among visible states or among hidden states, thus the name "restricted." RBM was previously demonstrated to work well for the Netflix challenge [20]. We implemented RBM using Matlab, and converted it to Pmode. For a model with 100 hidden units, it takes about 1 hour for one iteration without Pmode; using Pmode with 30 labs, it takes 3 minutes for one iteration.

The k-nearest neighbor (kNN) method is also popular for prediction. With a properly defined distance metric, for each data point needed for prediction, the weighted average of the ratings of its k closest neighbors is used to predict the rating of this point. Since there are so many user-user pairs for us to handle in reasonable time and space, we use a simplified approach with only movie-movie similarities. Again, we parallelize kNN by partitioning users into user groups so that each lab processes one user group.

For RBM itself, a score of 0.9181 is obtained. For kNN with $k = 21$ and a good similarity function, a RMSE of 0.9270 is obtained. Linear blending of ALS with kNN and RBM yields a RMSE of 0.8952 (ALS + kNN + RBM), and it represents a 5.91% improvement over Netflix's CineMatch system.

5 Related Work

There is a lot of academic and industrial work on recommendation systems, low-rank matrix approximation, and the Netflix prize. In the following we briefly discuss related work most relevant to ours.

[6] The experiment with $n_f = 1000$ is technically challenging as U takes $2G$ memory with single precision entries for each processor. We managed to run the procedure of updateM in two batches, while in each batch only two processors for each server are active and U is only replicated in these processors. This avoids memory thrashing using the 6G shared memory for each server. And ALS-WR converges in 10 rounds while each rounds takes 1.5 hours.

5.1 Recommendation Systems

Recommendation systems can be mainly categorized into content-based and collaborative filtering, and are well-studied in both academia and industry [16, 2, 7]. *Content-based* recommendation systems analyze the content (e.g., texts, metadata, features) of the item to identify related items, with exemplary systems InfoFinder [12], NewsWeeder [14]. *Collaborative Filtering* uses aggregated behavior/taste of a large number of users to suggest relevant items to specific users. Recommendations generated by CF are based solely on the user-user and/or item-item similarities, with exemplary systems GroupLens [19] and Bellcore Video Recommender [11]. Efforts to combine both content-based approach and collaborative filtering include the Fab system [3] and unified probabilistic framework [18].

5.2 The Netflix Prize Approaches

For the Netflix prize, Salakhutdinov et al. [20] used Restricted Boltzmann Machines (RBM), obtaining an RMSE score of slightly below 0.91. They also presented a low-rank approximation approach using gradient descent. Their low-rank approximation obtained an RMSE score slightly above 0.91 using between 20-60 hidden features. [7] The objective function of their SVD approach is the same as our ALS-WR, however we use alternating least squares instead of gradient descent to solve the optimization problem, and we are able to use a much large number of features (1000 vs 40) to obtain significant improvement in RMSE score.

Among many other approaches to the Netflix problem, Bell et al. [5] proposed a neighborhood-based technique which combines k-nearest-neighbor (kNN) and low-rank approximation to obtain significantly better results compared to either technique alone. Their team won the progress prize in October 2007, obtaining an RMSE score on the qualifying dataset of 0.8712, improving the CineMatch score by 8.5%. However, their solution [4] is a linear combination of 107 individual solutions, while multiple solutions are derived by variants of three classes of solutions (ALS, RBM, and kNN). For ALS alone, their best result was obtained using 128 hidden features with an RMSE score above 0.9000. For a comprehensive treatment of various approaches for the Netflix prize, see the individual papers presented in KDD Cup & Workshop 2007 [21, 17, 15, 13, 23].

5.3 Low-Rank Approximation

When a fully specified matrix is to be approximated by a low-rank matrix factorization, variants of singular value decomposition are used, for example in information retrieval (where SVD techniques are known as latent semantic indexing [9]). Other matrix factoring methods, for example nonnegative matrix factorization and maximum margin matrix factorization have also been proposed for the Netflix prize [23].

[7] We obtained similar results and got 0.9114 with $n_f = 50$.

For a partially specified matrix, the SVD is not applicable. To minimize the sum of squared differences between the known elements and the corresponding elements of the factored low rank matrix, ALS has proven to be an effective approach. It provides non-orthogonal factors, unlike SVD. The SVD can be computed one column at a time, whereas for the partially specified case, no such recursive formulation holds. An advantage of ALS is its easy parallelization. Like Lanczos for the sparse, fully specified case, ALS preserves the sparse structure of the known matrix elements and is therefore storage-efficient.

6 Concluding Remarks

We introduced a simple parallel algorithm for large-scale collaborative filtering which, in the case of the Netflix prize, performed as well as any single method reported in the literature. Our algorithm is designed to be scalable to very large datasets. Moderately better scores can be obtained by refining the RBM and kNN implementation or using more complicated blending schemes. ALS-WR in particular is able to achieve good results without using date or movie title information. The fast runtime achieved through parallelization is a competitive advantage for model building and parameter tuning in general. It will be interesting to develop a theory to explain why ALS-WR never overfits the data.

As the world shifts into Internet computing and web applications, large-scale data intensive computing becomes pervasive. Traditional single-machine, single-thread computing is no longer viable, and there is a paradigm shift in computing models. Parallel and/or distributed computing becomes an essential component for any computing environment. Google, the leading Internet company, is building its own proprietary parallel/distributed computing infrastructure, based on MapReduce [8], Google File System [10], Bigtable [6], etc. Most technology companies do not have the capital and expertise to develop an in-house large-scale parallel/distributed computing infrastructure, and prefer instead to use readily available solutions to solve computing infrastructure problems. Hadoop [1] is an open-source project sponsored by Yahoo!, which tries to replicate the Google computing infrastructure with open-source development. We have found parallel Matlab to be flexible and efficient, and very straightforward to program. Thus, from our experience, it seems to be a strong candidate for widespread, easily scalable parallel/distributed computing.

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